VC-dimension of neural networks

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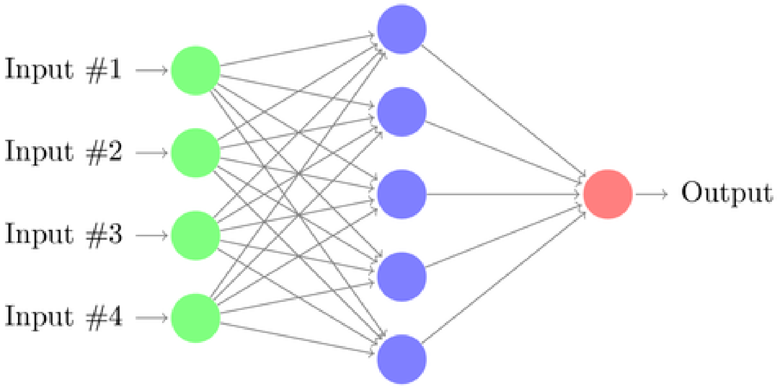
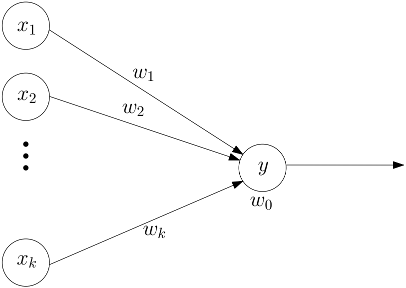
joint with Nick Harvey and Chris liaw
University of British Columbia

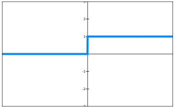
Neural networks



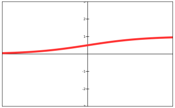
Neural networks



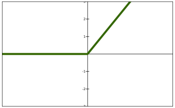
$$y = \sigma(w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_k x_k)$$



threshold



sigmoid



ReLU

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Theorem (fundamental theorem of statistical learning Blumer, Ehrenfeucht, Haussler, Warmuth'89)

The number of samples required for PAC learning within error ε in a model with VC-dimension v is essentially $\Theta(v/\varepsilon)$.

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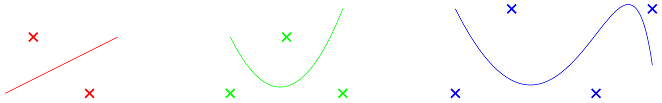
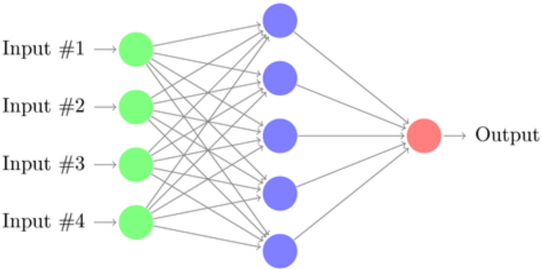
e.g. VC-dimension of polynomials of degree d is $d+1$.



VC-dimension of neural networks



$v(e, \ell) :=$ maximum VC-dimension of a neural network

e edges

ℓ layers

VC-dimension of neural networks

If the activation function is piecewise polynomial,

$$v(e,\ell) \le Ce^2 \qquad \text{[Goldberg, Jerrum'95]}$$

$$ce\ell \le v(e,\ell) \le C(e\ell^2 + e\ell\log e) \qquad \text{[Bartlett, Maiorov, Meir'98]}$$

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Theorem (Harvey, Liaw, M'16)

If the activation function is piecewise linear,

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GoogleNet'14: $\ell = 41$, $e = 7$ million